Boundary-Layer Considerations in the Design of Aerodynamic Contractions

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Most design methods for aerodynamic contractions are based on an inviscid approach even though the flow quality downstream of the contraction is determined primarily by viscous effects, including possible boundary-layer separation. The present study includes the consideration of boundary-layer behavior by application of the Stratford criterion for turbulent separation to numercially computed wall-pressure distributions. Calculations are presented for a four-parameter family of contractions within which the minimum-length contraction shapes consistent with fully attached boundary-layer flow are determined.

Introduction

THE principal concern of design methods for aerodynamics contractions is to produce a steady, parallel, and uniform exit flow. This can be accomplished, to a large degree, by avoiding flow separation through the alleviation of adverse pressure gradients.

It can be shown that whenever a converging duct segment is attached to constant-area segments at its inlet and exit, regions of adverse pressure gradient will occur along the wall near each end, possibly causing boundary-layer separation.[1] If separation occurs, it will act to degrade flow uniformity and steadiness, both of which are essential in a test facility. Separation is usually avoided if the adverse pressure gradients are minimized by making the contraction sufficiently long.

Some contraction design schemes consist merely of fitting a high-degree polynomial curve between two constant-area segments a given distance apart to yield a desired contraction ratio.[2,3] More sophisticated methods derived from fluid-mechanical considerations are based on inviscid techniques and involve determination of wall contours corresponding to specified axial velocity or acceleration distributions.[4-18] Attention to the boundary-layer separation problem takes the form of requiring a monotonic velocity increase in the streamwise direction or a nonnegative acceleration at every location between inlet and exit planes. The majority of these schemes result in contours of infinite length which must be arbitrarily truncated. Finite length designs are usually associated with physically nonrealistic requirements of flow uniformity at contraction inlet and exit planes. In either case, addition of constant-area segments at both ends of the contour invalidates the assumed velocity (pressure) distribution upon which the design is based, especially in the regions of the inlet and exit where separation is most likely to occur in an actual contraction. Qualitative arguments and subjective judgment have usually been necessary to finalize a particular design.

The present study carries contraction design methodology one step beyond that of previous investigators by including quantitative considerations of boundary-layer behavior. Pressure distributions on the walls of a four-parameter family of axisymmetric contractions have been computed. These have then been tested against the turbulent boundary-layer separation criterion developed by Stratford,[19] and minimum-length contractions which do not exhibit turbulent separation have been determined. The term "minimum-length" implies the shortest member of the family of contours considered here; it is conceivable that contour shapes yielding even shorter lengths could be found.

Received May 24, 1973; revision received May 3, 1974. This research was conducted under the McDonnell Douglas Independent Research and Development Program. The author appreciates the comments and suggestions of M. Sajben.

Index category: Nozzle and Channel Flow.

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Contraction Geometry

The axisymmetric duct configuration considered in the present study is shown in Fig. 1. The duct consists of three segments:† a constant-area inlet with radius R_i and length equal to its radius; a contraction of length L with area ratio $c = A_i/A_e$ and contour $R(x)$ as explained below; and a constant-area exit with radius $R_e = c^{-1/2}R_i$ and length equal to R_i. The lengths of inlet and exit segments have been determined by computer experiment to be such that any further increase in length will cause no appreciable change in the flow within the contraction or the constant-area regions immediately adjacent to the contraction.

A convenient means to generate a family of finite-length contractions is to specify the average streamwise acceleration distribution $f(x)$ in the convergent segment.[6] Assuming quasi-one-dimensional flow, the governing equations are

$$u(du/dx) = f(x) \tag{1}$$

$$uA = \text{constant} \tag{2}$$

where u denotes streamwise velocity, A is cross-sectional area, and x is distance along the duct centerline. The acceleration function is required to satisfy the conditions $f(x_i) = f(x_i + L) = 0$ and $f(x) \geq 0$ $[x_i \leq x \leq (x_i + L)]$. Equation (1) can be integrated and combined with Eq. (2) to obtain the following relations for the contraction contour $R(x)$:

$$\left(\frac{R_i}{R}\right)^4 = (c^2 - 1)\frac{F(x)}{F(x_i + L)} + 1[x_i \leq x \leq (x_i + L)] \tag{3}$$

where

$$F(x) = \int_{x_i}^{x} f(x)dx \tag{4}$$

The particular acceleration function chosen for the present study is

$$f(x) = \left\{\frac{1}{2}\left[1 - \cos 2\pi\left(\frac{x - x_i}{L}\right)^n\right]\right\}^p \tag{5}$$

†Subscripts i and e refer to inlet and exit stations, respectively.

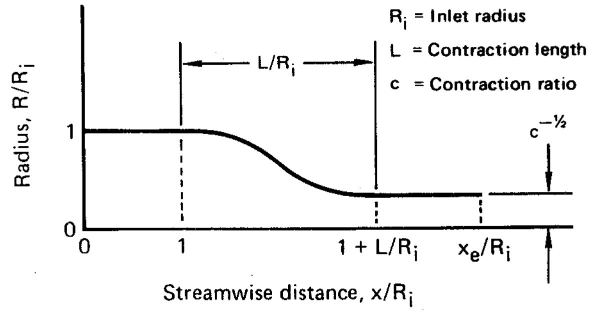

Fig. 1 Axisymmetric contraction geometry and nomenclature.

Equations (3-5) define a four-parameter family of contours determined by the contraction ratio c, the contraction length L, and the two exponents n and p. Figures 2-4 show representative curves of the family. In Fig. 4, the constant-area segments have been omitted.

Potential Flow

Inviscid, incompressible, irrotational, axisymmetric flow is governed by the following equation for the stream function:

$$\frac{\partial^2\psi}{\partial x^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial x^2} = 0 \tag{6}$$

$\psi(x,r)$ satisfies the continuity equation exactly and is related to velocity components $u(x,r)$ and $v(x,r)$ in the x and r directions, respectively, by the relations

$$ru = \frac{\partial\psi}{\partial r}, \quad rv = -\frac{\partial\psi}{\partial x}. \tag{7}$$

Boundary conditions for the configuration shown in Fig. 1 are as follows: At the inlet plane, the axial velocity component is specified to be constant, $u(0,r) = U_i$, so

$$\psi(0,r) = \frac{1}{2}r^2U_i. \tag{8}$$

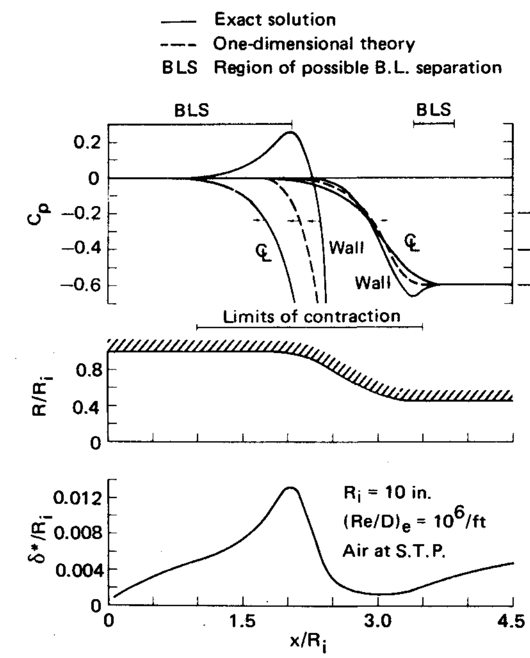

Fig. 2 Variation of pressure coefficient and turbulent boundary-layer displacement thickness in an unseparated 5:1 contraction with L/D_i = 1.25, n = 4.0, p = 0.8.

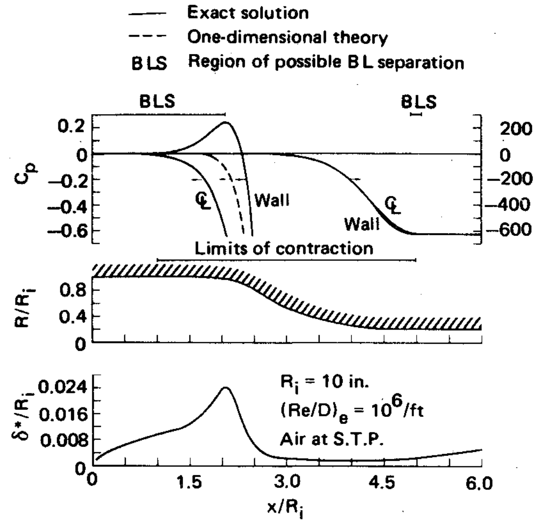

Fig. 3 Variation of pressure coefficient and turbulent boundary-layer displacement thickness in an unseparated 25:1 contraction with L/D_i = 2.0, n = 4.0, p = 0.8.

Although the elliptic nature of Eq. (6) prohibits the imposition of an additional parallel-flow condition $v = 0$ at the inlet, this is effectively accomplished by making the constant-area inlet segment of the duct sufficiently long. At the exit plane, the parallel-flow condition is easier to apply numerically than the condition of flow uniformity, so we set

$$\frac{\partial\psi}{\partial x}(x_e/R_i, r) = 0. \tag{9}$$

Both centerline and wall coincide with streamlines of the flow; hence,

$$\psi(x,0) = 0, \quad \psi(x,R) = \frac{1}{2}R_i^2U_i. \tag{10}$$

The pressure coefficient based on inlet conditions is given by

$$C_p = \frac{p - p_i}{\frac{1}{2}\rho U_i^2} = 1 - \frac{1}{r^2U_i^2}\left[\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial r}\right)^2\right]. \tag{11}$$

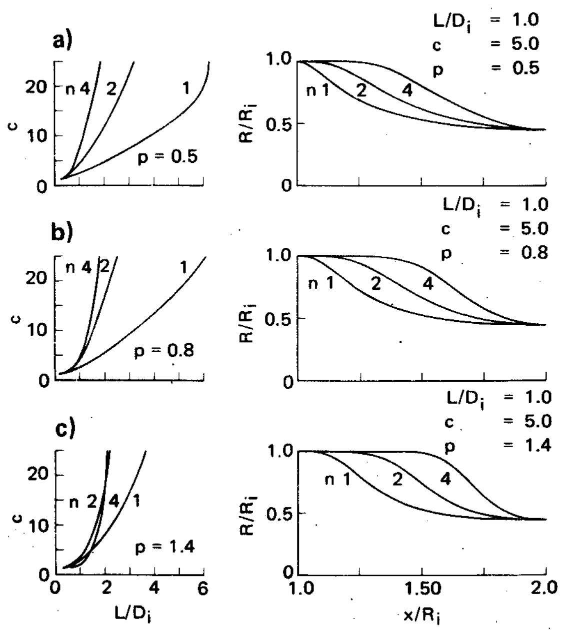

Fig. 4 Separation boundaries for a series of configuration families.

Equation (6) with boundary conditions (8–10) has been numerically integrated using a finite-difference, line-overrelaxation scheme developed by Hoffman.[20] Figures 2 and 3 show representative C_p distributions along the centerline and wall of contraction contours defined in the preceding section with $c = 5$ and 25, respectively. For comparison, the C_p distribution predicted by one-dimensional theory is also shown. Note that the influence of the contraction extends approximately one inlet radius into the constant-area segment located upstream and to a lesser degree into the constant-area segment downstream. Available numerical results indicate that the extent of downstream influence diminishes with increasing contraction ratio for the family of contours studied. Two regions of adverse pressure gradient appear along the duct wall, one near the inlet plane of the convergent segment and the other near the exit plane. A characteristic feature of the flow through the duct is the velocity peak which appears on the centerline in the constant-area inlet segment and persists far into the convergent segment where a reversal occurs and a centerline defect in velocity appears. The defect continues until approximately one exit radius downstream of the contraction exit plane where a nearly uniform velocity profile occurs again. The nonuniformities are easily explained by the required streamline curvatures, the associated radial gradients of static pressure, and the fact that the total pressure is constant for all radii. For $c = 5$, the centerline velocity defect relative to the wall velocity at the exit station has been found to be less than 0.25% of the average exit velocity; for $c = 25$, the centerline defect is less than 0.01%. These values have been obtained for all contours of the family studied here independent of length.

Test for Boundary-Layer Separation

Once the potental flow solution for the converging duct is available, an assessment of boundary-layer separation is possible. One approach is to perform a detailed computation of boundary-layer growth. Preliminary efforts along this line were made using the axisymmetric version of the Cebeci-Smith method.[21] Typical results for the turbulent boundary-layer displacement thickness δ^* on the duct wall are shown in Figs. 2 and 3 for the case of $R_i = 10$ in. and exit Reynolds number based on exit diameter $(Re/\mathrm{D})_e = 10^6/\mathrm{ft}$. Note the rapid boundary-layer growth in the constant-area inlet segment and the substantial reduction in thickness which occurs within the contraction itself. The exit Reynolds number was standardized because exit (test section) conditions would normally be specified in contraction design. In starting each boundary-layer computation, the duct inlet was treated like a leading edge, and transition from laminar to turbulent behavior was taken to occur at $x = 0.15\ R_i$. These starting conditions constitute an arbitrary choice and are considered to be representative of conditions that might exist just aft of the last screen in a wind-tunnel plenum chamber.

Because of the complexity and time-consuming set-up requirements of the Cebeci-Smith method, the simpler alternative of the Stratford turbulent separation criterion[19] was chosen for the present purposes. Although developed for two-dimensional boundary layers, the expected small ratio of boundary-layer thickness to contraction inlet radius justifies its application here. The Stratford criterion gives separation results in close agreement with the Cebeci-Smith method but tends to be conservative from a design standpoint by predicting separation to occur slightly earlier, i.e., further upstream.[22] It can be incorporated into the final stages of a potential flow computer program with little effort.

The previously stated exit condition used for this study corresponds to Reynolds numbers which fall below the range originally considered by Stratford. It was therefore necessary to remove certain numerical approximations included in his final equation. The form of the separation criterion used here was as follows:

$$\frac{(n+1)^{(n+1)/4}(n+2)^{1/2}}{(n-2)^{(n-2)/4}} C_p^{(n-2)/4}\left(x\frac{dC_p}{dx}\right)^{1/2} = 11.3217\,\beta(10^{-6}Re_x)^{1/10} \qquad (12)$$

where $n = \log_{10} Re_x$ and $\beta = 0.66$ or 0.73 for $d^2C_p/dx^2 < 0$ or $d^2C_p/\mathrm{dx}^2 \geq 0$, respectively. The distance x in Eq. (12) represents arc length along the wall measured from the beginning of the upstream constant-area section.

As given by Eq. (12), the separation criterion provides a valid test only in the region of adverse pressure gradient near the contraction inlet. When a favorable pressure gradient precedes the region of adverse gradient, the distances appearing in Eq. (12) must be measured from a virtual origin.[20] Since the family of contours used in the present study did not exhibit any tendency to separate near the contraction exit, as verified by selective applications of the Cebeci-Smith method, this modification was not used. In view of the tendency of contractions to cause a considerable reduction in boundary-layer thickness, as indicated by Figs. 2 and 3, there is reason to believe that inlet separation will usually be the dominant factor. Nevertheless, it is conceivable that a family of contours could be found with performance controlled by exit separation, in which case the modified interpretation of the separation criterion mentioned above would be applicable.

The assumption that the incident boundary layer is turbulent dictates the character of separation in the contraction inlet region of the present study; it will be a turbulent separation. In cases where separation near the exit becomes the determining factor, it may be necessary to consider laminarization of the turbulent boundary layer in the contraction. Experimental evidence indicates that laminarization can occur if the contraction inlet Reynolds number is sufficiently low.[23] Since the location of laminarization will always be downstream of any inlet separation point, this phenomenon is relevant only to exit separation. The effect can easily be accommodated in a design effort by switching from the turbulent separation criterion to its laminar counterpart, also due to Stratford,[24] at the appropriate streamwise wall station.

Results and Conclusions

Potential flow and boundary-layer separation calculations have been carried out for a series of contractions with contours defined by Eqs. (3–5). Figure 5 illustrates details of minimum length determination for contours with exponents $n = 4.0$ and $p = 0.8$. At fixed contraction ratio c, the length-to-inlet diameter ratio L/D_i is varied in increments of 0.25 to bracket the value at which separation occurs. This is done for a number of area ratios in the range $2 \leq c \leq 25$ thereby establishing a "separation boundary" for that particular subfamily of contours as indicated by the curved line.

Figures 4a–4c are plots of the separation boundaries for contractions defined by the nine possible combinations of exponents $n = 1, 2, 4$ and $p = 0.5, 0.8, 1.4$. Also shown are sample contours for $c = 5$ and $L/D_i = 1.0$. At the lower values of exponent p, the contraction length required to avoid flow separation is reduced as exponent n increases. This corresponds to a decrease of curvature at the contraction inlet and an increase of curvature near the exit plane for this particular family of contours. The above trend is progressively diminished as p increases, especially at smaller contraction ratios, and a reversal is evident for $p = 1.4$.

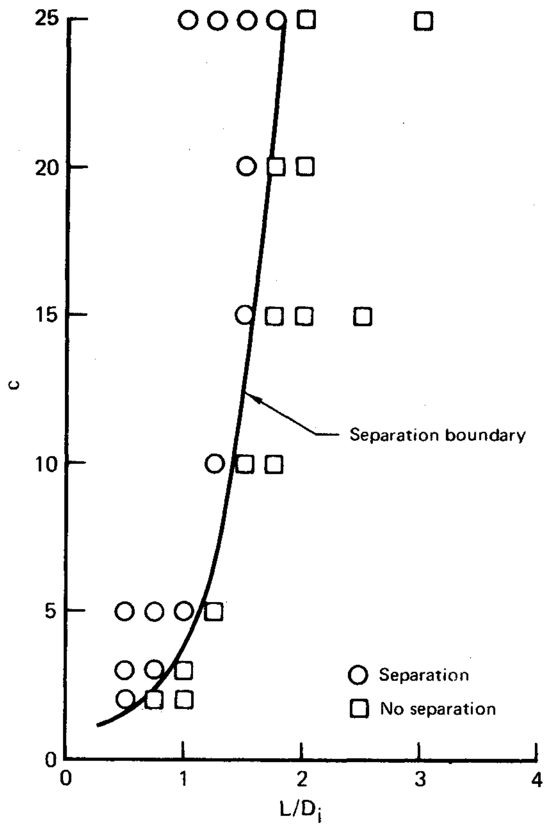

Fig. 5 Separation line determination for the contraction family corresponding to $n = 4.0$, $p = 0.8$ with $(Re/D)_e = 10^6$/ft, $R_i = 10$ in.

These results can be explained in terms of wall pressure distributions and the related boundary-layer behavior, as exhibited in Figs. 2 and 3, for example. Smaller inlet curvatures produce a milder C_p rise along the wall approaching the contraction and thus a slower boundary-layer growth. The corresponding contractions can therefore be made shorter before the adverse pressure gradient becomes severe enough to cause separation. However, too gradual an area reduction at the inlet, as in the case of the $n = 4$, $p = 1.4$ curve of Fig. 4c, tends to have a detrimental effect by subjecting the boundary layer to a relatively mild pressure gradient over a long distance.

The displacement thickness distributions shown in Figs. 2 and 3 give some indication why, in a contraction, inlet separation is probably more critical than exit separation. Adverse pressure gradients cause substantial boundary-layer growth in the low Reynolds number flow at the inlet enhancing the possibility of separation. After the flow has been accelerated by the contraction, however, the boundary layer is reduced considerably in thickness and is better able to withstand any adverse gradients which occur in the vicinity of the exit.

We conclude that the consideration of viscous effects is an important aspect of contraction design, and that it may be accomplished in a relatively simple way through application of the Stratford separation criterion. The present study has focused on a family of contractions dominated by inlet separation; however, the Stratford criterion also can be applied without inherent difficulty to designs in which exit separation constitutes the limiting performance factor.

References

[1]Bradshaw, P. and Pankhurst, R. C., "The Design of Low-Speed Wind Tunnels," NPL Aero. Rept. 1039, Sept. 1962, Aeronautical Research Council, London, England.

[2]Maestrello, L., "UTIA Air Duct Facility for Investigation of Vibration Noise Induced by Turbulent Flow Past a Panel (Boundary Layer Noise)," Tech. Note 20, April 1958, Institute of Aerophysics, Univ. of Toronto, Toronto, Canada.

[3]Hanson, C. E., "The Design and Construction of a Low-Noise, Low-Turbulence Wind Tunnel," Tech. Rept. 79611-1, Jan. 1969, Acoustics and Vibrations Lab., MIT, Cambridge, Mass.

[4]Tsien, H. S., "On the Design of the Contraction Cone for a Wind Tunnel," *Journal of Aeronautical Sciences,* Vol. 10, No. 2, Feb. 1943, pp. 68–70.

[5]Szczeniowski, B., "Contraction Cone for a Wind Tunnel," *Journal of Aeronautical Sciences,* Vol. 10, No. 8, Oct. 1943, pp. 311–312.

[6]Batchelor, G. K. and Shaw, F. S., "A Consideration of the Design of Wind Tunnel Contractions," Australian Rept. ACA-4 (ARC 7306), March 1944, Aeronautical Research Council, London, England.

[7]Hughes, N. J. S., "Stream Expansion with Discontinuity in Velocity on the Boundary," R & M 1978, March 1944, Aeronautical Research Council, London, England.

[8]Cheers, F., "Note on Wind-Tunnel Contractions," R & M 2137, March 1945, Aeronautical Research Council, London, England.

[9]Lighthill, M. J., "A New Method of Two-Dimensional Aerodynamic Design," R & M 2112, April 1945, Aeronautical Research Council, London, England.

[10]Thwaites, B., "On the Design of Contractions for Wind Tunnels," R & M 2278, March 1946, Aeronautical Research Council, London, England.

[11]Whitehead, L. G., Wu, L. Y., and Waters, M. H. L., "Contracting Ducts of Finite Length," *Aeronautical Quarterly,* Vol. 2, Pt. 4, Feb. 1951, pp. 254–271.

[12]Jordinson, R., "Design of Wind Tunnel Contractions," *Aircraft Engineering,* Vol. 33, No. 392, Oct. 1961, pp. 294–297.

[13]Cohen, M. J. and Ritchie, N. J. B., "Low-Speed Three-Dimensional Contraction Design," *Journal of the Royal Aeronautical Society,* Vol. 66, No. 616, April 1962, pp. 231–236.

[14]Lau, W. T. F., "An Analytical Method for the Design of Two-Dimensional Contractions," *Journal of the Royal Aeronautical Society,* Vol. 68, No. 637, Jan. 1964, pp. 59–62.

[15]Bossel, H. H., "Computation of Axisymmetric Contractions," *AIAA Journal,* Vol. 7, No. 10, Oct. 1969, pp. 2017–2020.

[16]Cohen, M. J. and Nimery, D. A., "Axisymmetric Nozzles for Compressible Flow," TAE Rept. 122, April 1971, Israel Institute of Technology, Haifa, Israel.

[17]Yuska, J. A., Diedrich, J. H., and Clough, N., "Lewis 9- by 15-Foot V/STOL Wind Tunnel," TM X-2305, July 1971, NASA.

[18]Barger, R. L. and Bowen, J. T., "A Generalized Theory for the Design of Contraction Cones and Other Low-Speed Ducts," TN D-6962, Nov. 1972, NASA.

[19]Stratford, B. S., "The Prediction of Separation of the Turbulent Boundary Layer," *Journal of Fluid Mechanics,* Vol. 5, Pt. 1, Jan. 1959, pp. 1–16.

[20]Hoffman, G. H., "Rotational Inviscid Flow in Axisymmetric Ducts," Rept. MDC Q0472, Dec. 1972, McDonnell Douglas Corp., St. Louis, Mo.

[21]Cebeci, T. and Smith, A. M. O., "A Finite-Difference Method for Calculating Compressible Laminar and Turbulent Boundary Layers," *Journal of Basic Engineering,* Vol. 92, No. 3, Sept. 1970, pp. 523–535.

[22]Cebeci, T., Mosinskis, G. J. and Smith, A. M. O., "Calculation of Separation Points in Incompressible Turbulent Flow," *Journal of Aircraft,* Vol. 9, No. 9, Sept. 1972, pp. 618–624.

[23]Back, L. H., Cuffel, R. F., and Massier, P. F., "Laminarization of a Turbulent Boundary Layer in Nozzle Flow," *AIAA Journal,* Vol. 7, No. 4, April 1969, pp. 730–733.

[24]Stratford, B. S., "Flow in the Laminar Boundary Layer near Separation," R & M 3002, Nov. 1954, Aeronautical Research Council, London, England.